NONSTEADY MOTION OF A DROPPING COMPRESSIBLE LIQUID IN PIPES UNDER DIFFERENT LAWS OF FRICTION

M. Ya. Kurakina, V. P. Radchenko, and V. A. Yufin

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Certain cases of nonsteady motion of a dropping compressible liquid with different laws of friction in conduits (in particular, in main oil pipelines) are considered. Results are presented of a solution of a number of problems of unsteady motion of a dropping liquid in pipes with a square law of resistance and for regimes characteristic of hydraulically smooth pipes, obtained numerically by the method of finite differences (method of nets). A comparison is made to results obtained by linearizing the term containing the square-law friction according to a previous [1] technique. Deviations of the parameters of nonsteady flows for given laws of resistance are studied.

We will consider a system of equations for one-dimensional nonsteady motion of a dropping compressible liquid in pipes under a previous [1] formulation,

$$\frac{\partial(\rho w)}{\partial t}+\frac{\partial}{\partial x}(p+\rho w^2)+\frac{\lambda}{2D}\rho w|w|=0;\ \frac{1}{c^2}\frac{\partial p}{\partial t}+\frac{\partial(\rho w)}{\partial x}=0,\quad 0<x<L,\quad t>0, \tag{1}$$

where p, ρ, and w are cross-sectionally averaged pressure, density, and liquid flow rate, λ is the hydraulic resistance coefficient, D is pipeline diameter, x is a coordinate along the pipe axis, t is a time variable, L is the length of the pipeline section, $c=\sqrt{K/\rho_0}$ is the speed of sound in the liquid, where $K=K_f/[1+a_1(K_f/E)]$ is the reduced compressibility modulus taking into account the elasticity of the pipe walls, K_f is the compressibility modulus of the liquid, and a_1 is a dimensionless coefficient depending on the shape of the cross-section and the thickness of the walls.

In the case of thin walls $a_1=D/\sigma_0$, where D is the interior pipe diameter, δ_0 is the thickness of the pipe wall, E is Young's modulus of the material of the pipe, and ρ_0 is density at a pressure p_0.

We propose that the dynamic equation of [1] be linearized in the following way in order to avoid difficulties in solving the system (1), due, in particular, to the nonlinearity of the term containing friction in this equation,

$$\frac{\lambda\rho w|w|}{2D}\approx\left(\frac{\lambda w}{2D}\right)_{\text{av}}\rho w=2a\rho w. \tag{2}$$

Here $2a=\lambda(w_2+2w_1)/3D$ and w_1 and w_2 are the ranges of variation of velocity w in given nonsteady motion.

The system of equations (1), linearized in accordance with (2), will be used also to calculate nonsteady flow of oil in main oil pipelines, though the flow regime there corresponds to turbulent flow in the zone of hydraulically smooth pipes. The term $(\partial/\partial x)/(\rho w^2)$ characterizing the variation of the velocity head throughout the length of the pipe is usually disregarded in the dynamics equation, i.e., we consider in place of Eq. (1) the linear system [1]

$$\frac{\partial(\rho w)}{\partial t}+\frac{\partial p}{\partial x}+2a\rho w=0; \tag{3}$$

$$\frac{1}{c^2}\frac{\partial p}{\partial t}+\frac{\partial(\rho w)}{\partial x}=0.$$

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Numerical solution on the BÉSM-6 computer of a number of problems of nonsteady flow of a liquid were obtained to estimate the error arising in this simplification of the initial system of equations. These equations were found based on the system of equations (1) within the zone characterized by square-law resistance and for turbulent flow of dropping liquids in the zone of hydraulically smooth pipes, at which $\lambda = 0.3164/\mathrm{Re}^{0.25}$ (Blasius zone). These numerical solutions were compared to the solution of Eqs. (3), linearized in accordance with [1] and also numerically obtained on a computer. The dependence of liquid density on pressure was assumed to follow the Hooke law,

$$\rho = \rho_0\left(1 + \frac{p - p_0}{K_f}\right).$$

The difference of pressure $p - p_0$ was assumed to be small in comparison with E and K_f. We assumed a stationary distribution of pressure and velocity along the pipeline as the initial conditions,

$$p(x) = p_0 - (p_0 - p_L)\frac{x}{L};\quad w(x) = w_0 = \mathrm{const}\ (0 \leqslant x \leqslant L), \tag{4}$$

where p_0 and p_L are pressure at the front and end of the pipe, respectively. Two types of boundary conditions were considered:

$$w(0,\ t) = f_1,\ p(L,\ t) = f_2; \tag{5}$$

$$p(0,\ t) = f_3,\ p(L,\ t) = f_4. \tag{6}$$

The method of nets was used to numerically solve the systems of equations (1) and (3) with initial and boundary conditions of the type of Eqs. (4)-(6). The characteristic form of Eqs. (1) was used in the numerical calculation,

$$\frac{\partial p}{\partial t} + (w \pm c)\frac{\partial p}{\partial x} \pm \rho c\left[\frac{\partial w}{\partial t} + (w \pm c)\frac{\partial w}{\partial x}\right] = \pm c\Phi, \tag{7}$$

where

$$\Phi = -\frac{\lambda}{2D}\rho w|w|,$$

and correspondingly, for the system (3),

$$\left(\frac{\partial p}{\partial t} \pm c\frac{\partial p}{\partial x}\right)(c \pm w) \pm \rho c^2\left(\frac{\partial w}{\partial t} \pm c\frac{\partial w}{\partial x}\right) = \pm c^2\Phi_1, \tag{8}$$

where

$$\Phi_1 = -2a\rho w$$

The following finite-difference scheme [2] was used to solve the system of equations (7):

$$(p_{i,k+1} - p_{i,k})/\tau + (w \pm c)_{i,k}(p_{i+1,k+1} - p_{i-1,k+1})/2h$$
$$\pm(\rho c)_{i,k}[(w_{i,k+1} - w_{i,k})/\tau + (w \pm c)_{i,k}(w_{i+1,k+1} - w_{i-1,k+1})] = c\Phi_{i,k},\quad i = 1,\ 2, \ldots,\ N-1;$$
$$(p_{0,k+1} - p_{0,k})/\tau + (w_{0,k} - c)(p_{1,k+1} - p_{0,k+1})/h - (\rho c)_{0,k}[(w_{0,k+1} - w_{0,k})/\tau + (w_{0,k} - c)(w_{1,k+1} - w_{0,k+1})/h] = -c\Phi_{0,k};$$
$$(p_{N,k+1} - p_{N,k})/\tau + (w_{N,k} + c)(p_{N,k+1} - p_{N-1,k+1})/h + (\rho c)_{N,k}$$
$$\times[(w_{N,k+1} - w_{N,k})/\tau + (w_{N,k} + c)(w_{N,k+1} - w_{N-1,k+1})/h] = c\Phi_{N,k}.$$

An analogous implicit difference scheme was also used to solve the system of equations (8). The methods of solving systems of quasilinear hyperbolic equations using the characteristic form have been considered in [3].

The resulting closed system of linear algebraic equations was solved by the pivot method, the systems of equations (7) and (8) being first reduced to dimensionless form by the introduction of the variables $\bar{x} = x/L$; $\bar{p} = p/p_0$; $\bar{w} = w/w_0$; and $\bar{t} = t/t_0$, where $t_0 = L/c$.

Results of calculations of nonsteady flow of a dropping liquid, for which we may assume in Eqs. (1) and (3) that $\rho = \rho_0 = \mathrm{const}$ practically without any error [1], are presented as an example.

The resulting numerical solutions made stable and made to converge by experimentally checking the rectangular net as the pitches were varied. Calculations were carried out under the boundary conditions

$$w(0,\ t)=2w_0,\ p(L,\ t)=p_L; \tag{9}$$

$$w(0,\ t)=3w_0,\ p\ (L,\ t)=p_L; \tag{10}$$

$$p(0,\ t)=2p_0,\ p\ (L,\ t)=p_L. \tag{11}$$

The values of the initial parameters were [4] $L=109$ km, $D=0.509$ m, $p_0=32.25\cdot10^4$ kg/m^2, $p_L=2.61\cdot10^4$ kg/m^2, $w_0=1$ m/sec, $c=1100$ m/sec,* $\lambda=0.0266$, $\rho=88.8$ kg$\cdot$sec^2/m^4, and $\nu=0.25\cdot10^4$m^2/sec, where ν is kinematic viscosity.

A comparison of the results of calculations of the linearized system of equations (3) to solutions of the nonlinear system of equations (1) for square-law friction and conditions characterized by hydraulically smooth pipes is depicted in Figs. 1-4 [boundary conditions (9)] and in Figs. 5-8 [boundary conditions (11)], in which curves 1 correspond to the linearized system of equations, curves 2, to the zone of hydraulically smooth pipes, and curves 3, to the zone of square-law friction. Figure 1 implies that the function $\bar{w}(\bar{x},\ \bar{t})$ at the start of the intermediate process ($\bar{t}=0.3$) sharply decreases in magnitude on the segment $0<\bar{x}<0.5$ and then remains practically constant and equal to $0.5\ \bar{w}(0,\ \bar{t})$ (zone of hydraulically smooth pipes and zone of square-law friction) or $0.25\ \bar{w}(0,\ \bar{t})$ (linearized equations). Velocity constantly increases, and the divergence between the velocity values for different laws of friction decreases after substantial periods of time.

The intermediate process ceases throughout all of $\bar{w}=2\bar{w}_0$ when $\bar{t}=11.7$ for the Blasius zone, $\bar{t}=12.0$ for the linearized system of equations, and $\bar{t}=15.0$ for the zone of square-law friction.

The variation of pressure with respect to time under the boundary conditions (9) is depicted in Fig. 2. Pressure in the pipeline increases with time and by $\bar{t}=1.5$ doubles at the front of the pipeline for the zone of hydraulically smooth pipes and for the square-law friction and reaches values $\bar{p}(0,\ \bar{t})=2.74$ for the linearized system of equations. The velocity–time distribution for three pipeline cross sections is depicted in Fig. 3 under the boundary conditions (9). Figure 4 implies that the pressure throughout the entire intermediate process, calculated using the linearized equations, is significantly greater than for the zone of hydraulically smooth pipes or for square-law friction. The variation of the relative deviations of velocity and pressure with time for the linearized equations is compared in Table 1 [boundary conditions (9)] to the case of square-law friction and the regime of hydraulically smooth pipes. The subscripts "l", "s", and "B", refer to the case of a linearized system, square-law friction, and continuous friction (Blasius zone).

The nature of the variation of velocity and pressure are as before with an increase in disturbing action [boundary conditions (10)], though the time of the intermediate process increases ($\bar{t}=13.5$ for the Blasius zone, $\bar{t}=13.8$ for the linearized system of equations and $\bar{t}=19.5$ for square-law friction).

Results of numerical calculations of nonsteady flows under the boundary conditions (9) and (10) have demonstrated that the linearized system of equations yields understated values of the velocity (cf. Figs. 1 and 3) and overstated values of the pressure (cf. Figs. 2 and 4) in comparison with conditions characterized by hydraulically smooth pipes. For example, the maximal relative deviations in velocity under the boundary

*The speed of sound was given by $\sqrt{K/\rho_0}=c=1080$ m/sec under the initial data we have assumed [$K_f=1.4\cdot10^8$ kg/m^2 and $E=2\cdot10^{10}$ kg/m^2 (steel), and $\delta=0.01$ m]. The calculations used an experimentally obtained [4] value $c=1100$ m/sec close to the above value.

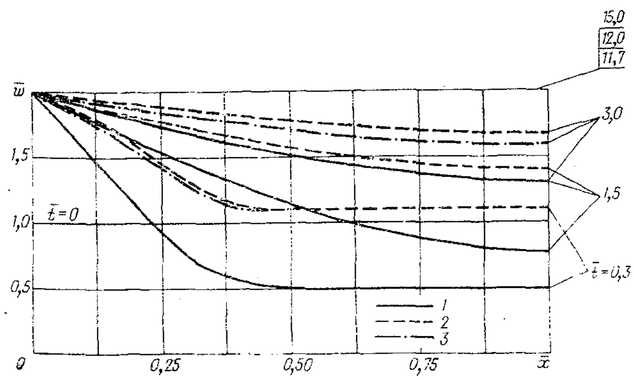

Fig. 1

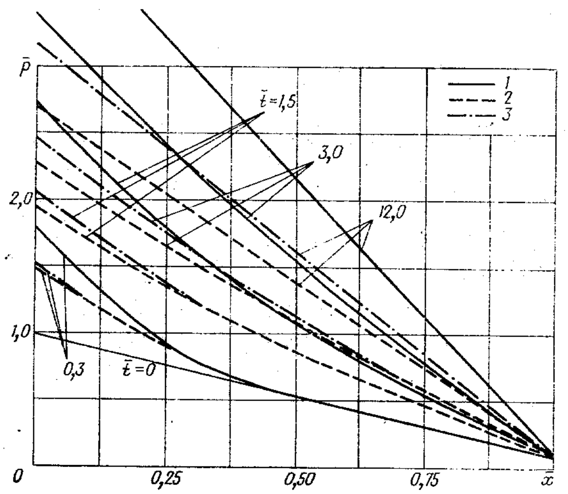

Fig. 2

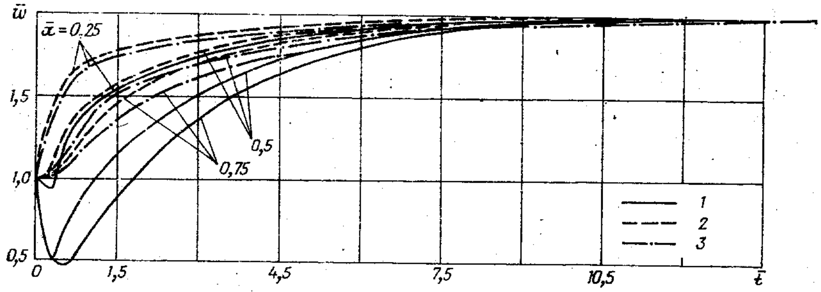

Fig. 3

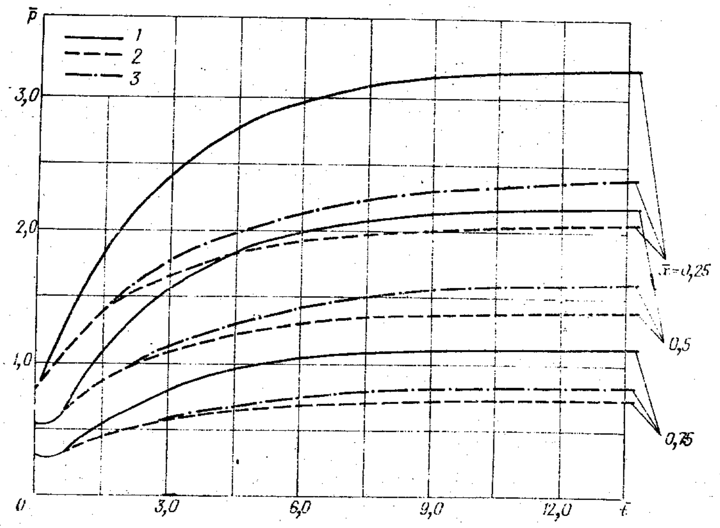

Fig. 4

TABLE 1

$\bar{t}$	$\bar{x}=0,25$					$\bar{x}=0,5$					$\bar{x}=0,75$				
	$\bar{w}_l$	$\bar{w}_s$	$\bar{w}_B$	δw_s	δw_B	$\bar{w}_l$	$\bar{w}_s$	$\bar{w}_B$	δw_s	δw_B	$\bar{w}_l$	$\bar{w}_s$	$\bar{w}_B$	δw_s	δw_B
0	1,00	1,00	1,00	0,0	0,0	1,00	1,00	1,00	0,0	0,0	1,00	1,00	1,00	0,0	0,0
0,3	0,94	1,41	1,45	33,4	35,2	0,52	1,03	1,09	51,8	52,3	0,50	1,07	1,08	52,7	53,6
0,6	1,29	1,60	1,65	19,4	21,8	0,71	1,28	1,34	44,4	48,0	0,47	1,10	1,12	57,4	58,0
1,2	1,49	1,70	1,74	12,4	14,4	1,05	1,46	1,51	28,0	30,4	0,75	1,30	1,36	42,3	44,9
2,1	1,63	1,78	1,82	8,4	10,4	1,32	1,60	1,67	17,5	21,0	1,10	1,49	1,57	26,2	29,9
3,0	1,74	1,83	1,88	4,93	7,45	1,31	1,70	1,77	11,2	14,7	1,37	1,62	1,70	15,4	19,4
6,0	1,92	1,93	1,96	0,52	2,04	1,85	1,88	1,93	1,6	4,15	1,80	1,84	1,91	2,18	5,76
12,0	2,00	1,99	2,00	0,45	0,00	1,99	1,98	2,00	0,81	0,1	1,99	1,98	2,00	1,01	[illegible]
	$\bar{p}_l$	$\bar{p}_s$	$\bar{p}_B$	δp_s	δp_B	$\bar{p}_l$	$\bar{p}_s$	$\bar{p}_B$	δp_s	δp_B	$\bar{p}_l$	$\bar{p}_s$	$\bar{p}_B$	δp_s	δp_B
0	0,77	0,77	0,77	0,0	0,0	0,54	0,54	0,54	0,0	0,0	0,31	0,31	0,31	0,0	0,0
0,3	0,93	0,90	0,90	3,34	3,34	0,54	0,54	0,54	0,0	0,0	0,31	0,31	0,31	0,0	0,0
0,6	1,22	1,06	1,06	15,1	15,1	0,65	0,63	0,64	3,18	1,56	0,32	0,32	0,32	0,0	0,0
1,2	1,63	1,29	1,27	26,4	28,4	0,96	0,80	0,81	20,0	18,5	0,46	0,41	0,43	12,2	7,0
2,1	2,09	1,55	1,5	34,8	39,4	1,31	0,99	0,98	32,4	33,7	0,66	0,51	0,51	29,4	29,4
3,0	2,41	1,75	1,66	37,8	45,0	1,54	1,13	1,10	36,3	40,0	0,79	0,59	0,58	33,9	36,2
6,0	2,97	2,13	1,92	39,5	54,7	1,98	1,42	1,30	39,4	52,3	1,02	0,74	0,68	37,8	50,0
12,0	3,21	2,38	2,05	30,6	56,8	2,17	1,61	1,39	34,5	55,6	1,13	0,85	0,74	33,1	52,7

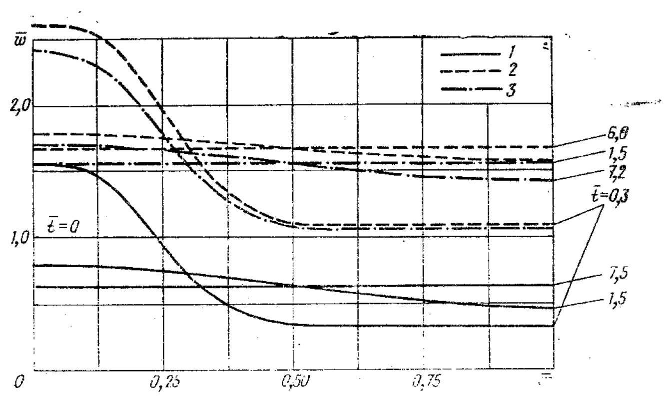

Fig. 5

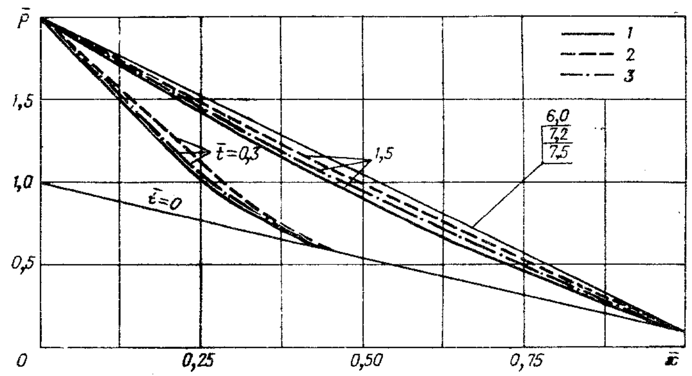

Fig. 6

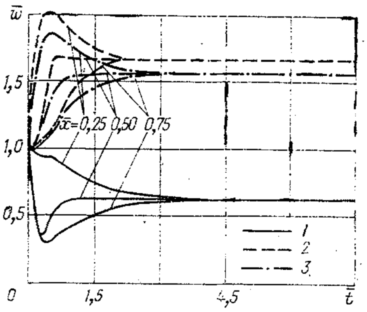

Fig. 7

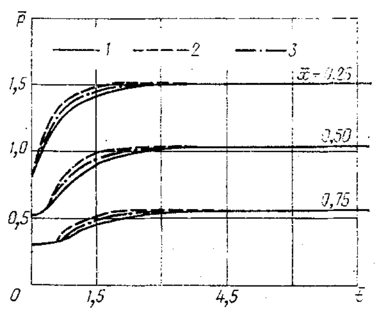

Fig. 8

conditions (9) amounted to $\delta|_{\bar{t}=0.3}=35.2$, $\delta|_{\bar{t}=0.3}=52.3$, and $\delta|_{\bar{t}=0.6}=58$ (in percent), respectively, for the three pipeline cross sections (x = 0.25, 0.5, and 0.75). The maximal relative deviations increased and amounted to (in percent) $\delta|_{\bar{t}=0.3}=39.2$, and $\delta|_{\bar{t}=0.3}=62.7$, and $\delta|_{\bar{t}=0.6}=67$ under the boundary conditions (3.2).

The relative pressure deviations for the same pipeline cross sections at moment of time $\bar{t}=9.0$ were constant and were approximately 50-55% for the boundary conditions (9) (cf. Table 1) and 45-50% for the boundary conditions (10). The velocity–time distribution for the boundary conditions (11) are depicted in Fig. 7, which implies that the function $\bar{w}(\bar{x}, \bar{t})$ at the start of the intermediate process sharply increases for the Blasius zone and square-law friction and decreases for the linearized system of equations. Velocity becomes constant at moment of time $\bar{t}=3.0$ for square-law friction $[\bar{w}(\bar{x}, \bar{t})=1.56]$ and for the Blasius zone $[\bar{w}(\bar{x}, \bar{t})=1.67]$, while $\bar{w}(\bar{x}, \bar{t})=0.62$ for the linearized system at moment of time $\bar{t}=4.5$.

The pressure distribution relative to the pipeline length is depicted in Fig. 6 under the boundary conditions (11). Pressure in the pipeline increases with increasing time and at $\bar{t}=6.0$ the intermediate process ceases for the regime of hydraulically smooth pipes, at $\bar{t}=7.2$ for square-law friction, and at $\bar{t}=7.5$ for the linearized system of equations. It should be noted that the pressures and velocities obtained using the linearized system of equations under the boundary conditions (11) are below the corresponding values for the Blasius regime and for square-law friction.

Our results imply that the linearized equations for nonsteady motion of a liquid in pipelines that we have used introduces a substantial error in determining the pressure (45-55%) and velocity (40-70%). The maximal divergence of the corresponding results of the calculations using equations for the Blasius regime and for square-law friction is at most 10% for velocity and 12% for pressure.

LITERATURE CITED

1. I. A. Charnyi, Nonsteady Motion of an Actual Fluid in Pipes [in Russian], Gostekhteorizdat (1951).
2. A. S. Vladislavlev, B. M. Pisarevskii, V. M. Pisarevskii, and V. P. Radchenko, "Gasdynamic calculation of pulsating flow in pipelines," Zh. Prikl. Mekh. Tekh. Fiz., No. 4, 85-88 (1972).
3. O. F. Vasil'ev, S. K. Godunov, et al., "Numerical method of calculating propagation of long waves in open river channels and its application to the problem of floods," Dokl. Akad. Nauk SSSR, 151, No. 3, 525-527 (1963).
4. P. A. Moroz, N. F. Galitskov, and B. M. Prokhorov, "Experimental studies of hydrodynamical processes in a pipeline," in: Transport and Storage of Petroleum and Petroleum Products [in Russian], No. 6, Tsentr. Nauchnoz Issled. Inst. Tekhnol. Eneftegaz (1964).